Published by Institute of Physics Publishing for SISSA/ISAS

Received: *January 3, 2003*
Accepted: *February 17, 2003*

Confining phase superpotentials for SO/Sp gauge theories via geometric transition

Hiroyuki Fuji and Yutaka Ookouchi

Department of Physics, Tokyo Institute of Technology
Tokyo 152-8511, Japan
E-mail: hfuji@th.phys.titech.ac.jp ookouchi@th.phys.titech.ac.jp

Abstract: We examine a large N duality via geometric transition for $\mathcal{N}=1$ SO/Sp gauge theories with superpotential for adjoint chiral superfield. In this paper, we find that the large N gauge theories are exactly analyzed for the classical quartic superpotentials by the finite rank SO/Sp gauge theories. With this classical superpotentials, we evaluate the confining phase superpotentials using the Seiberg-Witten theory. In the dual theory, we calculate the superpotential generated by the R-R and NS-NS 3-form fluxes. As the non-trivial examples, we discuss for SO(6), SO(8) and Sp(4) gauge theories. In these cases we have the perfect agreement of the confining phase superpotentials up to the 4th order of the glueball superfields.

Keywords: Duality in Gauge Field Theories, String Duality.

JHEP02(2003)028

Contents

1. Introduction

It has been conjectured that the large N gauge theory describes the string theory [1]. The most prominent example of this large N duality is AdS/CFT correspondence [2]. As an example, type-IIB string theory on $AdS_5 \times T^{1,1}$ realizes $\mathcal{N} = 1$ supersymmetric gauge theory with conformal symmetry in large N limit, and the field theory results are reproduced from the supergravity [3]. In [4], the breaking of the conformal symmetry of this model can be discussed by introducing the fractional branes.

As a large N duality for topological string theory, *gauge theory/geometry* correspondence is proposed [5]. Via conifold transition, this duality claims that the topological string theory on the resolved conifold is equivalent to the topological string theory on the deformed conifold with N A-branes wrapped on the special lagrangian 3-cycle in the large N limit.

This topological string duality is extended to the large N duality for the superstring theory [6] and M-theory [7, 8]. For type-IIB string theory, T-duality reverses the direction of the transition [6, 9]. The duality claims that the string theory on the resolved conifold with N D5-branes wrapped on the exceptional $\mathbf{P}^1$ is equivalent to that on the deformed conifold with N units of 3-form fluxes through the special lagrangian 3-cycle S^3 in large N limit. Here the fluxes on the defomed conifold generate the superpotential and break the supersymmetry spontaneously [10]. Therefore, an $\mathcal{N} = 2$ vector multiplet splits into an $\mathcal{N} = 1$ chiral and a vector multiplet. Especially, this chiral superfield is identified with the glueball superfield in the confining phase of the gauge theory realized on the N D5-branes in the large N limit. Under this identification, the superpotential generated by 3-form fluxes on the deformed conifold coincides with that of the massive glueball superfield [11] in the gauge theory. In this way, the validity of the duality is examined for this geometry.

Applying the conifold transition locally, the duality is also considered for more complicated geometries [12]–[14]. In the resolved geometry, classical superpotential for U(N) adjoint chiral superpotential arises in the gauge theory on N D5-branes wrapping on the exceptional $\mathbf{P}^1$'s, and it leads to $\mathcal{N}=1$ supersymmetric theory. After the geometric transition, n exceptional $\mathbf{P}^1$'s shrink and n S^3's are replaced in the dual theory. Thus the dual theory is defined on the deformed geometry. On this dual geometry, 3-form fluxes appear on S^3's after the transition and generate superpotential. When the above duality conjecture is applied, the superpotential in the dual theory is also identified with the effective one for the original gauge theory in large N limit. For some examples, the confining phase superpotentials for both theories coincides perfectly [12].

By introducing orientifold projection, the conifold is resolved by $\mathbf{P}^1$ wrapped orientifold plane and the gauge group becomes SO(N)/ Sp(N). The large N duality of unoriented string is examined for the topological string theory [15, 16] and type-IIB superstring theory [17]–[19]. In [17], the geometric transition for SO(N) gauge theory is discussed in this set up. As in U(N) gauge theory case, the world-volume theory of N D5-branes with orientifold plane wrapped on $\mathbf{P}^1$ realize SO(N)/ Sp(N) gauge theory with arbitrary classical superpotential for the adjoint chiral superfield. In order to confirm the duality proposal for this case, it is also necessary to examine the coincidence of the physical quantities on both theories. In [17], it is found that the effective coupling constant of SO(N) gauge theory agrees with that of the dual theory.

In order to examine the duality for SO / Sp gauge theories further, we evaluate the confining phase superpotentials on both theories in this paper. On the gauge theory side, we need to evaluate it in large N limit of SO(N)/ Sp(N) gauge group. We prove that it can be evaluated exactly from the *finite* rank gauge theory for the SO / Sp gauge theory with the classical quartic superpotential for adjoint chiral superfield. On the dual theory side, we evaluate the periods for the deformed geometry with the orientifolding. By identifying the periods with the expectation values of the gluon superfields, we obtain the effective superpotentials. To evaluate the superpotentials explicitly on both sides, we consider SO(6), SO(8) and Sp(4) gauge theories as the non-trivial examples. As a result, we find the perfect agreement of these superpotentials up to the 4th order of the glueball superfields.

This paper is organized as follows. In section 2, we will review the geometric transition [6] and see how the gauge theory and the string theory are exactly analyzed [12, 17]. In section 3, we will prove that the confining phase superpotential for SO($2N$)/ Sp($2N$) gauge theory in the large N limit is exactly evaluated from the finite rank gauge theory with the classical quartic superpotential. And then we will compute the confining phase superpotential using the Seiberg-Witten theory for the gauge groups SO(6), SO(8) and Sp(4). In section 4, we will explicitly analyze the dual geometry and evaluate the effective superpotential from the computation of the periods. By comparing the results of section 3 and section 4, we will find the coincidence of the exact superpotentials. In the appendix we will show the detailed computations of periods in section 3.

2. Geometric transition and large N duality

2.1 The geometric transition

We consider type-IIB string theory on non-compact Calabi-Yau manifold,

$$M_s : W'(x)^2 + y^2 + z^2 + w^2 = 0 \,, \tag{2.1}$$

where $W(x)$ is defined as,

$$W(x) \equiv \sum_{p=1}^{n+1} \frac{g_p}{p} x^p \,. \tag{2.2}$$

This Calabi-Yau manifold is resolved by locating $\mathbf{P}^1$'s at the singularities where $W'(x) = 0$ is satisfied. The resolution is given by the surface in $\mathbf{C}^4 \times \mathbf{P}^1$.

$$\begin{pmatrix} y + iz & w + iW'(x) \\ -w + iW'(x) & y - iz \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \,, \tag{2.3}$$

where λ_1, λ_2 are coordinate of $\mathbf{P}^1$. When N D5-branes wraps on the exceptional $\mathbf{P}^1$'s and fill the flat 4 dimensional space-time, 4 dimensional $\mathcal{N} = 1$ supersymmetric $\mathrm{U}(N)$ gauge theory with superpotential W_{cl} for the adjoint chiral superfield Φ,

$$W_{\mathrm{cl}}(\Phi) = \sum_{p=1}^{n+1} \frac{g_p}{p} \mathrm{Tr}\Phi^p \,, \tag{2.4}$$

is realized [20].

In order to realize $\mathrm{SO}(N)/\,\mathrm{Sp}(N)$ gauge theories, we introduce orientifold projection which is complex conjugation [19, 34]. For singular geometry (2.1) this projection acts as

$$(x, y, z, w, \lambda_1, \lambda_2) \to (-x, -y, -z, -w, \lambda_1, \lambda_2) \,. \tag{2.5}$$

In this projection λ_1, λ_2 are invariant then orientifold plane wraps $\mathbf{P}^1$ at $x = y = z = w = 0$. Remaing Minkowski space also fixed the orientifold plane is $O5$-plane.

In this geometry world-volume theory on the D5-brane is $\mathrm{SO}\,/\,\mathrm{Sp}$ gauge theory with the following tree level superpotential [17],

$$W_{\mathrm{cl}}(\Phi) = \sum_{p=1}^{n+1} \frac{g_{2p}}{2p} \mathrm{Tr}\Phi^{2p} \equiv \sum_{p=1}^{n+1} g_{2p} u_{2p} \,, \,, \tag{2.6}$$

where Φ is the chiral superfield in the adjoint representation of $\mathrm{SO}(N)/\,\mathrm{Sp}(N)$ gauge group and $u_{2p} \equiv \frac{1}{2p}\mathrm{Tr}\Phi^{2p}$.

We define parameters a_i by

$$W'(x) = \sum_{p=1}^{n} g_{2p} x^{2p-1} = g_{2n+2}\, x \prod_{i=1}^{n} (x^2 + a_i^2) \,. \tag{2.7}$$

In the classical vacua of this gauge theory, the eigenvalues of Φ become roots $0, \pm i a_i$'s of $W'(x) = 0$. When N_0 D5-branes wrap on $\mathbf{P}^1$ at $x = 0$ and N_i D5-branes wrap on the $\mathbf{P}^1$ located at $x = \pm i a_i$, the vacuum of the gauge theory becomes classically $P(x) \equiv \det(x - \Phi) = x^{N_0} \prod_{i=1}^{n} (x^2 + a_i^2)^{N_i}$ and the gauge group breaks as,

$$\mathrm{SO}(N) \to \mathrm{SO}(N_0) \otimes_{i=1}^{n} \mathrm{U}(N_i)\,, \qquad \mathrm{Sp}(N) \to \mathrm{Sp}(N_0) \otimes_{i=1}^{n} \mathrm{U}(N_i)\,, \tag{2.8}$$

where $N = N_0 + \sum_{i=1}^{n} N_i$. This theory can be analyzed non-perturbatively in terms of the Seiberg-Witten theory [21, 22] and we will review in the next section.

The large N dual of this theory is found via conifold transition [5, 6]. The conifold is defined as [23, 24],

$$x^2 + y^2 + z^2 + w^2 = 0\,. \tag{2.9}$$

This geometry has a singularity at $x = y = z = w = 0$. This singularity can be removed in two ways. One way is the deformation of Kähler structure. This singularity is blown up and replaced by $\mathbf{P}^1$. The resulting geometry becomes $\mathcal{O}(-1) \oplus \mathcal{O}(-1)$ bundle over $\mathbf{P}^1$, and this is called resolved conifold. Another way is the deformation of the complex structure, where the defining equation is deformed as,

$$x^2 + y^2 + z^2 + w^2 = \mu^2\,. \tag{2.10}$$

Thus the singularity is replaced by S^3 with the radius μ and resulting geometry becomes T^*S^3. This manifold is called deformed conifold. When N D5-branes are wrapped on $\mathbf{P}^1$ in the resolved conifold, D5-branes disappear and N units of R-R 3-form flux H_R through S^3 and NS-NS 3-form flux H_{NS} through the dual 3-cycle remain after the conifold transition. In the large N limit, these two theories are equivalent and give the same physical quantities [6]. After the conifold transition, D5-branes and orientifold plane disappear and $N \mp 2$ units[1] of R-R 3-form flux H_R through $\mathbf{S}^3$ and NS-NS 3-form flux H_{NS} through the dual 3-cycle remain .

In the case of the Calabi-Yau manifold (2.1), above analysis can be applied locally. Through the geometric transition, the dual Calabi-Yau manifold is defined by replacing all $\mathbf{P}^1$'s by S^3's. The deformed geometry is the following hypersurface in $\mathbf{C}^4$,

$$M_{\mathrm{cpx}}:\; g \equiv W'(x)^2 + f_{2n-2}(x) + y^2 + z^2 + w^2 = 0\,, \tag{2.11}$$

where $f_{2n-2}(x)$ is the degree $(n-1)$-th polynomial of x^2. In this deformed geometry M_{cpx}, we choose the integral basis of the 3-cycles A_i, B_i $(i = 1, \ldots, h_{2,1} = 2n+1)$ which satisfy the symplectic pairing,

$$(A_i, B_j) = -(B_j, A_i) = \delta_{ij}\,, \qquad (A_i, A_j) = (B_i, B_j) = 0\,, \tag{2.12}$$

where the pairing (A, B) of three-cycles A, B is defined as the intersection number. For the deformed Calabi-Yau manifold (2.11), these 3-cycles are constructed as $\mathbf{S}^2$ fibration over the line segments between two critical points $x = 0^+, 0^-, \pm i a_1^+, \pm i a_1^- \cdots \pm i a_n^+, \pm i a_n^-, \infty$ of $W'(x)^2 + f_{2n-2}(x)$ in x-plane. Therefore we set the three cycle A_0 to be the $\mathbf{S}^2$ fibration over the line segment between 0^- and 0^+ and three cycle A_i to be the fibration over the

[1]The sign $\mp$ is determined by the sign of the charge of orientifold plane.

JHEP02(2003)028

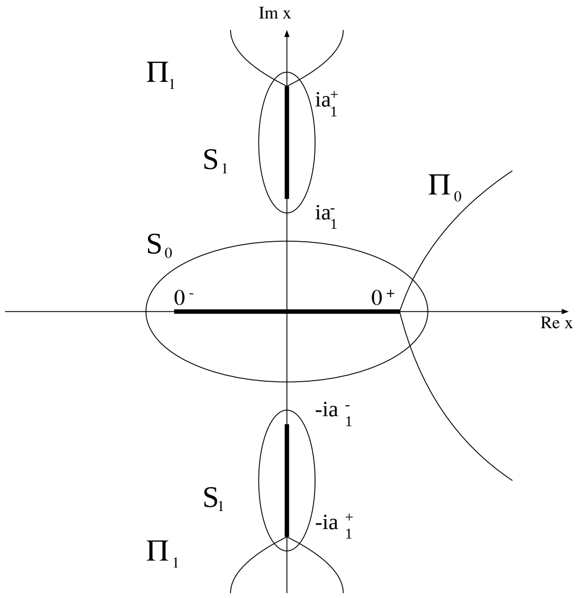

Figure 1: The 3-cycles which are constructed as the $\mathbf{S}^2$ fibration over the line segments in the complex plane.

line segment between ia_i^- and ia_i^+. On the other hand, three cycle B_0 is constructed as $\mathbf{S}^2$ fibration over the line segment between 0^+ and Λ_0 and three cycle B_i to be the fibration over the line segment between ia_i^+ and $i\Lambda_0$, Here we introduced the cut-off Λ_0, as these cycles are non-compact. Since this geometry has $\mathbb{Z}_2$ symmetry, the discussion is restricted to the upper half of x-plane in the following [17].

The holomorphic 3-form Ω for the deformed geometry (2.11) is given by

$$\Omega = 2\frac{dx \wedge dy \wedge dz}{\partial g / \partial w} \, . \tag{2.13}$$

The periods S_i and dual periods Π_i for this deformed geometry is given as,

$$S_i = \int_{A_i} \Omega \, , \qquad \Pi_i = \int_{B_i} \Omega \, . \tag{2.14}$$

The dual periods are expressed in terms of the prepotential $\mathcal{F}(S_i)$ such as,

$$\Pi_i = \frac{\partial \mathcal{F}}{\partial S_i} \, . \tag{2.15}$$

Since these 3-cycles are constructed as $\mathbf{S}^2$ fibrations, these periods are written in terms of the integrals over x-plane as,

$$\begin{aligned} S_0 &= \frac{1}{2\pi i}\int_{0^-}^{0^+} \omega \, , \qquad & S_i &= \frac{1}{2\pi i}\int_{ia_i^-}^{ia_i^+} \omega \, , \\ \Pi_0 &= \frac{1}{2\pi i}\int_{0^+}^{\Lambda_0} \omega \, , \qquad & \Pi_i &= \frac{1}{2\pi i}\int_{ia_i^+}^{i\Lambda_0} \omega \, , \end{aligned} \tag{2.16}$$

where ω is obtained by integrating holomorphic 3-form over the fiber $\mathbf{P}^1$,

$$\omega = 2dx \, \left(W'(x)^2 + f_{2n-2}(x)\right)^{1/2} \, . \tag{2.17}$$

2.2 Partial SUSY breaking and confinement of gauge theory

When the geometric transition occurs, the exceptional $\mathbf{P}^1$'s on which D5-branes wrap in the resolved geometry, is replaced by 3-form fluxes through the special lagrangian 3-cycles in the deformed geometry. This 3-form fluxes generate the superpotential, and $\mathcal{N} = 2$ supersymmetry for the dual theory is broken partially to $\mathcal{N} = 1$ supersymmetry [10]. In this subsection, we will review how the partial supersymmetry breaking occurs in the dual geometry and how the supermultiplets in the dual theory are identified with that of the effective gauge theory by the large N duality conjecture [6].

The partial supersymmetry breaking of $\mathcal{N} = 2$ theory to $\mathcal{N} = 1$ theory occurs by the electric and magnetic Fayet-Iliopoulous superpotential terms as [25],

$$W_{\mathrm{FI}} = \sum_i \int d\theta^2 d\tilde{\theta}^2 \left(e_i \Psi_i + m_i \frac{\partial \mathcal{F}}{\partial \Psi_i} \right), \tag{2.18}$$

where e_i's and m_i's are electric and magnetic charge respectively, and Ψ_i's are $\mathcal{N} = 2$ superfields and the holomorphic function $\mathcal{F}$ is the prepotential for the $\mathcal{N} = 2$ theory.

Turning on R-R and NS-NS fluxes through the special lagrangian 3-cycles, the above partial supersymmetry breaking is realized in type-IIB string theory [10]. The 3-form fluxes generate the superpotential [26]

$$-\frac{1}{2\pi i} W_{\mathrm{eff}} = \int \Omega \wedge (H_{\mathrm{R}} + \tau H_{\mathrm{NS}}), \tag{2.19}$$

where H_R and H_{NS} are 3-form fluxes and τ is the complexified type-IIB string coupling, and Ω is the holomorphic 3-form on the Calabi-Yau manifold. In the case of dual theory defined through geometric transition, H_{R} and H_{NS} satisfy,

$$N_0 \mp 2 = \int_{A_0} H_{\mathrm{R}}, \qquad N_i = \int_{A_i} H_{\mathrm{R}}, \qquad \alpha = \int_{B_i} H_{\mathrm{NS}}, \tag{2.20}$$

where α is the 4 dimensional bare gauge coupling constant g_0 as $\alpha \equiv 4\pi i / g_0^2$.

Plugging these relations into (2.19), the superpotential for the dual theory is expressed in terms of periods S_i and dual periods Π_i of the deformed Calabi-Yau manifold such as,

$$-\frac{1}{2\pi i} W_{\mathrm{eff}} = \left(\frac{N_0}{2} \mp 1 \right) \Pi_0 + \sum_{i=1}^{n} N_i \Pi_i + \alpha \sum_{i=0}^{n} S_i . \tag{2.21}$$

With this superpotential, $\mathcal{N} = 2$ vector multiplets Ψ_i splits into the massive $\mathcal{N} = 1$ chiral superfields S_i and massless $\mathrm{U}(1)^n$ vector multiplets. Following the large N duality proposal [6], massless $\mathrm{U}(1)^n$ vector multiplets are identified with those in the effective theory of the $\mathcal{N} = 1$ $\mathrm{SO}(N)/\,\mathrm{Sp}(N)$ gauge theory with the classical superpotential $W_{\mathrm{cl}}(\Phi)$. The $\mathcal{N} = 1$ massive chiral superfield S_i is identified with the glueball superfield,

$$S_i = -\frac{1}{32\pi^2} \mathrm{Tr}_{\mathrm{SU}(N_i)} W_\alpha W^\alpha , \tag{2.22}$$

where W_α is defined as $W_\alpha \equiv i\bar{D}^2 e^{-V} D_\alpha V$ with supercovariant derivative D_α and $\mathcal{N} = 1$

vector multiplet V. Thus the dual theory on the deformed geometry with fluxes corresponds to the confining phase of the gauge theory which is determined in terms of the resolved geometry.

To examine this correspondence, we should check that the low energy superpotential W_{exact} in the confining phase of $\mathcal{N}=1$ $\mathrm{SO}(N)/\mathrm{Sp}(N)$ gauge theory coincides with the superpotential $W_{\text{eff}}(S_i)$ which is generated by the deformed geometry with 3-form fluxes. In the following sections, we will evaluate these superpotentials and see their coincidences.

3. Confining phase superpotentials

3.1 Confining phase superpotentials for SO / Sp SYM

The confining phase of the pure $\mathcal{N}=2$ gauge theory is analyzed by the Seiberg-Witten theory [27]. For the gauge group $\mathrm{SO}(2N)$, the Seiberg-Witten curve is written as,

$$y^2 = \left[P_{2N}(x^2, u_i)\right]^2 - 4x^4\Lambda^{4N-4}\,, \tag{3.1}$$

where the characteristic polynomial P_{2N} is defined as [28],

$$\begin{aligned} P_{2N}(x^2, u_i) &\equiv \det(x - \Phi) = \sum_{k=0}^{2N} x^{2N-2k} s_{2k}, \\ \Phi &\equiv \operatorname{diag}\left(ia_1\sigma_2, ia_2\sigma_2, \ldots, ia_N\sigma_2\right). \end{aligned} \tag{3.2}$$

Here σ_a, $(a=1,\ldots,3)$ are the Pauli matrices and s_{2k}'s satisfy the following Newton's relations,

$$ks_{2k} + \sum_{r=1}^{k} ru_{2k}s_{2k-2r} = 0\,, \qquad u_k \equiv \frac{1}{k}\mathrm{Tr}\Phi^k\,. \tag{3.3}$$

For the gauge group $\mathrm{Sp}(2N)$, the Seiberg-Witten curve is written as [29, 30],

$$x^2y^2 = \left[P_{2N}(x^2, u_i) + 2\Lambda^{2N+2}\right]^2 - 4\Lambda^{4N+4}\,, \tag{3.4}$$

where the characteristic polynomial P_{2N} is defined for the adjoint superfield Φ which is defined as,

$$\begin{aligned} {}^t\Phi &= J\Phi J\,, \qquad J = \operatorname{diag}(i\sigma_2, \ldots, i\sigma_2)\,, \\ J\Phi &= \operatorname{diag}(a_1\sigma_1, \ldots, a_N\sigma_1)\,. \end{aligned} \tag{3.5}$$

When the superpotential $W_{\text{cl}}(\Phi)$ is introduced, the $\mathcal{N}=2$ gauge theory is deformed and the resulting theory has the unbroken supersymmetry on the submanifold of the Coulomb branch. This submanifold is determined by the locus where Seiberg-Witten curve degenerates. The monopoles or dyons become massless on some particular submanifold $\langle u_{2k}\rangle$. Near a point with l massless monopoles, the superpotential is,

$$W = \sum_{k=1}^{l} M_k(u_{2r})q_k\tilde{q}_k + \sum_{p=1}^{n+1} g_{2p}u_{2p}\,. \tag{3.6}$$

JHEP02(2003)028

On the supersymmetric vacua, $\langle u_{2k}\rangle$'s satisfy,

$$M_k(\langle u_{2k}\rangle) = 0\,, \qquad g_{2k} + \sum_{p=1}^{l} \frac{\partial M_p(\langle u_{2k}\rangle)}{\partial\langle u_{2k}\rangle}\langle q_p\tilde{q}_p\rangle = 0\,. \tag{3.7}$$

Therefore the superpotential in this vacuum is simply [21][22],

$$W_{\mathrm{exact}} = \sum_{p=1}^{n+1} g_{2p}\langle u_{2p}\rangle\,. \tag{3.8}$$

In the confining phase where $2N - 2n - 2$ monopoles become mutually local and massless, the SO($2N$) Seiberg-Witten curve has double zeros [17, 32] as,

$$\left[P_{2N}\left(x^2, u_i\right)\right]^2 - 4x^4\Lambda^{4N+4} = x^2\left[H_{2N-2n-2}\left(x^2\right)\right]^2 F_{4n+2}(x)\,. \tag{3.9}$$

For Sp($2N$) gauge group, $2N - 2n$ monopoles become massless and Seiberg-Witten curve has double zeros [31] as,

$$\left[P_{2N}\left(x^2, u_i\right) + 2\Lambda^{2N+2}\right]^2 - 4\Lambda^{4N+4} = \left[H_{2N-2n}\left(x^2\right)\right]^2 F_{4n}(x)\,. \tag{3.10}$$

Thus the exact superpotential in the confining phase is evaluated from the Seiberg-Witten curve with the massless monopole constraints (3.9) and (3.10).

3.2 Confining phase superpotential at large N

We have seen that the exact confining phase superpotential is evaluated from the Seiberg-Witten theory. Next we will consider its large N limit. In order to examine the duality, we need to evaluate the exact confining phase superpotential for SO(N) and Sp(N) gauge group in the large N limit. In this subsection, we will show that a solution for the massless monopole constraints (3.9) and (3.10) of the SO($2KN - 2K + 2$)/ Sp($2KN + 2K - 2$) gauge group is found from that of SO(N)/ Sp(N) gauge group via Chebyshev polynomials [33].

For the gauge group SO($2KN - 2K + 2$) with the classical superpotential $W_{\mathrm{cl}}(\Phi)$, the gauge group breaks in the classical vacuum as,

$$\mathrm{SO}(2KN - 2K + 2) \to \mathrm{SO}(2KN_0 - 2K + 2) \otimes_{i=1}^{n} \mathrm{U}(KN_i)\,, \tag{3.11}$$

where N_i's satisfy $N_0 + \sum_{i=1}^{n} N_i = N$. We choose $P_{2KN-2K+2}$ such as,

$$P_{2KN-2K+2}(x) = \widetilde{\Lambda}^{4KN-4K} x^2 T_K\left(\frac{P_{2N}(x)}{x^2\Lambda^{2N-2}}\right), \tag{3.12}$$

where Chebyshev polynomials $T_K(x)$ and $U_K(x)$ ($K = 0, 1, 2, \ldots$) are defined as,

$$\begin{aligned} T_K(x) &\equiv \cos(K \arccos x)\,, \qquad U_K(x) \equiv \sin(K \arccos x)\,, \\ T_K(x)^2 - 4 &= (x^2 - 4)U_{K-1}(x)^2\,. \end{aligned} \tag{3.13}$$

Then this $P_{2KN-2K+2}$ satisfies the massless monopole condition for $\mathrm{SO}(2KN-2K+2)$ gauge theory as,

$$\begin{aligned}
&P_{2KN-2K+2}(x)^2 - 4x^4\widetilde{\Lambda}^{4KN-4K} = \\
&= \widetilde{\Lambda}^{4KN-4K}\Lambda^{-4N+4}\left[U_{K-1}\left(\frac{P_{2N}(x)}{x^2\Lambda^{2N}}\right)\right]^2 \left(P_{2N}(x)^2 - 4x^4\Lambda^{4N-4}\right) \\
&\equiv x^2\left[H_{2KN-2n-2}(x)\right]^2 F_{4n+2}(x)\,.
\end{aligned} \tag{3.14}$$

Thus we found a solution of the massless monopole constraint for $\mathrm{SO}(2KN-2K+2)$ gauge theory.

For the gauge group $\mathrm{Sp}(2N)$, the exact superpotential can be analyzed in the same manner. In the classical vacuum, the gauge group $\mathrm{Sp}(2KN+2K-2)$ breaks in the classical vacuum as,

$$\mathrm{Sp}(2KN+2K-2) \to \mathrm{Sp}(2KN_0+2K-2)\otimes_{i=1}^{n}\mathrm{U}(KN_i)\,. \tag{3.15}$$

If we choose $P_{2KN+2K-2}(x)$ as,

$$P_{2KN+2K-2}(x) = \frac{\widetilde{\Lambda}^{2KN+2K}}{x^2}T_K\left(\frac{x^2P_{2N}(x)}{\Lambda^{2N+2}}+2\right) - 2\frac{\widetilde{\Lambda}^{2KN+2K}}{x^2}\,, \tag{3.16}$$

this satisfies the massless monopole constraint for the $\mathrm{Sp}(2KN+2K-2)$ gauge theory as follows.

$$\begin{aligned}
&\left(x^2P_{2KN+2K-2}(x)+2\widetilde{\Lambda}^{2KN+2K}\right)^2 - 4\widetilde{\Lambda}^{4KN+4K} = \\
&= \widetilde{\Lambda}^{4KN+4K}\Lambda^{-4N-4}\left[U_{K-1}\left(\frac{P_{2N}(x)^2}{\Lambda^{2N+2}}\right)\right]^2\left(\left(x^2P_{2N}(x)+2\Lambda^{2N+2}\right)^2 - 4\Lambda^{4N+4}\right) \\
&\equiv \left[H_{2KN-2n}(x)\right]^2 F_{4n}\,.
\end{aligned} \tag{3.17}$$

Thus a solution of the massless monopole constraint for $\mathrm{Sp}(2KN+2K-2)$ gauge theory is also expressed in terms of the Chebyshev polynomial.

To evaluate the exact superpotential, we need to find the $\langle\tilde{u}_k\rangle$ for $\mathrm{SO}(2KN-2K+2)/\,\mathrm{Sp}(2KN+2K-2)$ gauge theory. By expanding out (3.12), the vacuum expectation values $\langle\tilde{u}_k\rangle$ for $\mathrm{SO}(2KN-2K+2)/\,\mathrm{Sp}(2KN+2K-2)$ gauge theory are related with the vacuum expectation values $\langle u_k\rangle$ for $\mathrm{SO}(2N)/\,\mathrm{Sp}(2N)$ as,

$$\begin{aligned}
\tilde{u_2} &= Ku_2\,, \qquad \tilde{u_4} = Ku_4\,, \\
\tilde{u_6} &= Ku_6 + (K^2-K^3)\frac{u_2^3}{6},\ldots.
\end{aligned} \tag{3.18}$$

When we consider the quartic classical superpotential $W_{\mathrm{cl}}(x) = gx^4/4 + mx^2/2$, the exact superpotential for $\mathrm{SO}(2KN-2K+2)/\,\mathrm{Sp}(2KN+2K-2)$ gauge theory can be expressed in terms of that of $\mathrm{SO}(N)/\,\mathrm{Sp}(N)$ as,

$$W_{\mathrm{exact}}(\tilde{u}_i, g_i) = KW_{\mathrm{exact}}(u_i, g_i)\,. \tag{3.19}$$

For the completeness, we will consider the classical quartic superpotentials for $\mathrm{SO}(2KN -2K+2)/\mathrm{Sp}(2KN+2K-2)$ gauge theory. For the gauge group $\mathrm{SO}(2N)$ the classical quartic superpotential $W_{\rm cl}(\Phi)$ is evaluated in the classical vacuum as,

$$W_{\rm cl}^{\mathrm{SO}(2N)} = -\frac{N_1 m^2}{2g}\,. \tag{3.20}$$

For the gauge group $\mathrm{Sp}(2N)$, the classical quartic superpotential $W_{\rm cl}$ is given in the vacuum as,

$$W_{\rm cl}^{\mathrm{Sp}(2N)} = -\frac{N_1 m^2}{2g}\,. \tag{3.21}$$

Since the gauge groups break as (3.11) and (3.15) in the vacuum, the classical quartic superpotential $\widetilde{W}_{\rm cl}$ for the $\mathrm{SO}(2KN-2K+2)/\mathrm{Sp}(2KN+2K-2)$ gauge theory is also written as $\tilde{W}_{\rm cl} = KW_{\rm cl}$.

On the other hand, the factorization property of the effective superpotential in the deformed geometry is considered as follows. Before the geometric transition, $\mathrm{SO}(2KN-2K+2)$ gauge theory is realized by wrapping $2NK-2K+2$ D5-branes on $\mathbf{P^1}$ with orientifold plane[2] around the exceptional 2-cycles in the resolved geometry. Therefore the effective superpotential $W_{\rm eff}(S_i)$ generated by the 3-form flux in the dual theory is evaluated as,

$$\begin{aligned} W_{\rm eff}^{\mathrm{SO}(2NK-2K+2)} &= ((2N_0K-2K+2)-2)\,\Pi_0 + \sum_{i=1}^{n}(2KN_i)\Pi_i \\ &= KW_{\rm eff}^{\mathrm{SO}(2N)}\,. \end{aligned} \tag{3.22}$$

In the same way, the $\mathrm{Sp}(2KN+2K-2)$ gauge theory is realized by wrapping $2NK+2K-2$ D5-branes and a O^+-plane around the exceptional 2-cycles in the resolved geometry. The effective superpotential $W_{\rm eff}(S_i)$ in the dual theory is evaluated as,

$$\begin{aligned} W_{\rm eff}^{\mathrm{Sp}(2NK+2K-2)} &= ((2N_0K+2K-2)+2)\,\Pi_0 + \sum_{i=1}^{n}(2KN_i)\Pi_i \\ &= KW_{\rm eff}^{\mathrm{Sp}(2N)}\,. \end{aligned} \tag{3.23}$$

Thus the factorization property holds for the effective superpotential in the deformed geometry.

In this way, exact superpotential for $\mathrm{SO}(2KN-2K+2)/\mathrm{Sp}(2KN+2K-2)$ gauge theory with the quartic classical superpotential is expressed via that of $\mathrm{SO}(2N)/\mathrm{Sp}(2N)$ gauge theory. Using this analysis, we can discuss the large N exact superpotential by taking the limit $K\to\infty$ and the large N duality can be examined by checking the coincidence of the superpotentials for the *finite* rank gauge group.

[2]In this paper, we call O^--plane (resp. O^+-plane) as orientifold plane with negative (resp. positive) R-R charge.

3.3 Computation of confining phase superpotentials

In this subsection, we will evaluate the confining phase superpotentials for finite rank gauge groups in terms of the gauge theoretical analysis. Although the coincidence should be hold for any N, we will concentrate on some non-trivial examples as SO(6), SO(8) and Sp(4) gauge theories in this paper.

Case 1: $\mathrm{SO}(6) \to \mathrm{SO}(4) \times \mathrm{U}(1)$**.** In this case characteristic polynomial $P_6(x)$ which satisfies the constraint (3.9) is given by

$$P_6 = x^4(x^2 - b^2) - 2\Lambda^4 x^2 \,. \tag{3.24}$$

Using the formula (3.3), we obtain the following relations,

$$u_2 = b^2 \qquad u_4 = \frac{b^4}{2} + 2\Lambda^4 \,. \tag{3.25}$$

Thus the low energy superpotential is obtained as,

$$W_{\text{exact}} = \frac{m^2}{g}\left[\frac{1}{2}\left(\frac{gb^2}{m}\right)^2 + 2t^2 + \left(\frac{gb^2}{m}\right)\right]. \tag{3.26}$$

Integrating out b, we can get the exact superpotential,

$$W_{\text{exact}} = -\frac{m^2}{2g} + \frac{2m}{g}t^2 \,. \tag{3.27}$$

where $t \equiv g\Lambda^2/m$. The gauge symmetry breaking can be read off in the classical limit, $\Lambda \to 0$. Comparing the above result with (3.20), we find $N_1 = 1$.[3] Using the relation $2N = 2N_0 + 2N_1$ and $2N = 6$, we obtain $2N_0 = 4$. Thus we found the exact superpotentials corresponding to the breaking as $\mathrm{SO}(6) \to \mathrm{SO}(4) \times \mathrm{U}(1)$.

Case 2: Splitting of SO(8)**.** Similarly we will analyze the gauge group SO(8). In this case, we need to solve (3.9) for $2N = 8$,

$$P_8^2(x) - 4\Lambda^{12}x^4 = x^2\left[H_4(x)\right]^2 F_4(x) \,. \tag{3.28}$$

Let us set[4] $H_4(x) = x^2(x^2 - a^2)$ and $P_8(x) = x^8 + s_2x^6 + s_4x^4 + s_6x^2 + s_8$. The condition (3.28) gives us following relations,

$$s_8 = 0 \,, \qquad s_6^2 = 4\Lambda^{12} \,, \qquad s_4 = -3a^4 - 2s_2a^2 \,, \tag{3.29}$$

$$a^4(2a^2 + s_2) = \pm 4\Lambda^6 \,. \tag{3.30}$$

[3]In this subsection, we consider the brane configuration as $2N_0$ D5-branes wrapping on $\mathbf{P}^1$ with orientifold plane and N_1 D5-branes wrapping on each $\mathbf{P}^1$'s. Therefore we are considering the gauge symmetry breaking as $\mathrm{SO}(2N) \to \mathrm{SO}(2N_0) \times \mathrm{U}(N_1)$.

[4]Here we choose this particular ansatz for H_4 in order to avoid considering the gauge symmetry breaking as $\mathrm{SO}(8) \to \mathrm{SO}(2) \times \mathrm{U}(3)$. In the later discussion, the expansion parameter T for the effective superpotential of the deformed geometry is found to be ill-defined for this case.

Here we introduce new variable $b^2 \equiv 2a^2 + s_2$. Using this, we can rewrite (3.29) and (3.30) as,

$$s_4 = a^4 - 2a^2b^2 \,, \qquad a^4b^2 = \pm 4\Lambda^6 \,. \tag{3.31}$$

By the Newton's relation (3.3), the Casimirs are now found as,

$$u_2 = 2a^2 - b^2 \,, \qquad u_4 = a^4 + \frac{1}{2}b^4 \,, \tag{3.32}$$

and the low energy superpotential finally takes the form,

$$\begin{aligned} W_{\text{exact}} &= gu_4 + mu_2 + \beta(a^4b^2 \pm 4\Lambda^6) \\ &= \frac{m^2}{g}\left[x^4 + \frac{1}{2}y^4 + 2x^2 - y^2 + \gamma(x^4y^2 \pm 4t^3)\right], \end{aligned} \tag{3.33}$$

where β and $\gamma \equiv \beta m/g^2$ are Lagrange multipliers, and x, y are dimensionless variables defined by $a^2 = mx^2/g, b^2 = my^2/g$. To get the low-energy superpotential, we want to integrate out x, y. Therefore we have to solve $\partial W_{\text{exact}}/\partial x = 0, \partial W_{\text{exact}}/\partial y = 0$. Eliminating the Lagrange multipliers, we obtain the following equation and constraint for x, y.

$$\begin{aligned} x^4 + x^2 - y^4 + y^2 &= 0 \,, \\ x^4y^2 &= \pm 4t^3 \,. \end{aligned} \tag{3.34}$$

From the above relations, we can see how two different splittings will come out. In the classical limit $\Lambda \to 0$, the relations can be solved in two ways, namely, $x = 0$ or $y = 0$. Plugging these solutions into the superpotential, the former case corresponds to the gauge symmetry breaking $SO(6) \times U(1)$ and the latter case corresponds to that of $SO(4) \times U(2)$.

$SO(8) \to SO(6) \times U(1)$. First, we will consider the solution which become $x = 0$ in the classical limit. The equations (3.34) are rewritten as,

$$\frac{4t^3}{y^2} + \frac{2t^{3/2}}{y} - y^4 + y^2 = 0 \,. \tag{3.35}$$

This equation can be solved recursively using $t^{3/2}$ as expansion parameter. Plugging this solution into (3.33), we obtain the low energy superpotential for this case,

$$W_{\text{exact}} = \frac{m^2}{g}\left[-\frac{1}{2} + 4t^{3/2} + 2(t^{3/2})^2 - 2(t^{3/2})^3 + 4(t^{3/2})^4 - 21(t^{3/2})^5 + \mathcal{O}\left((t^{3/2})^6\right)\right]. \tag{3.36}$$

As $SO(6)$ case, we can read off the gauge symmetry breaking pattern from the classical limit of this potential as $SO(8) \to SO(6) \times U(1)$.

$SO(8) \to SO(4) \times U(2)$. Next, we will consider the solution which become $y = 0$ in the classical limit. In this case, the equations (3.34) are rewritten as,

$$x^4 + x^2 - \frac{16t^6}{x^8} - \frac{t^3}{x^4} = 0 \,. \tag{3.37}$$

JHEP02(2003)028

We can solve this equation as before but using as expansion parameter t^3. Plugging this back in $W_{\rm exact}$, we get the following superpotential.

$$W_{\rm exact} = \frac{m^2}{g}\left[-1 + 4t^3 - 8t^6 + 64t^9 - 768t^{12} + \mathcal{O}(t^{13})\right]. \tag{3.38}$$

In the classical limit, the gauge symmetry breaking pattern is found as, SO(8) $\to$ SO(4) $\times$ U(2).

Case 3: Sp(4) $\to$ Sp(2) $\times$ U(1). As an example of Sp($2N$) gauge theory, we will consider Sp(4) gauge group. The massless monopole condition (3.10) becomes in this case as,

$$(x^2P_4(x) + 2t^3)^2 - 4t^6 = (x^2 - a^2)^2F_8(x). \tag{3.39}$$

Let us set $P_4(x) = x^4 + s_2x^2 + s_4$. The above condition gives us following equations .

$$s_4 = -3a^4 - 2s_2a^2, \qquad a^4(2a^2 + s_2) = 4t^3. \tag{3.40}$$

Introducing new variable $b^2 \equiv 2a^2 + s_2$, the equation (3.40) is rewritten as,

$$s_2 = b^2 - 2a^2, \qquad s_4 = a^4 - 2a^2b^2, \qquad a^4b^2 = 4t^3. \tag{3.41}$$

Using the Newton's relation (3.3), we have the relations as,

$$u_2 = 2a^2 - b^2, \qquad u_4 = a^4 + \frac{b^4}{2}. \tag{3.42}$$

Under the constraint (3.39), the low energy superpotential is written as,

$$W_{\rm exact} = \frac{m^2}{g}\left[x^4 + \frac{y^4}{2} + 2x^2 - y^2 + \beta\left(x^4y^2 - 4t^3\right)\right], \tag{3.43}$$

where β is a Lagrange multiplier and x, y are dimensionless variable defined by $a^2 = mx^2/g, b^2 = my^2/g$. As SO($2N$) case, we have to integrate out x, y. After some calculations, we obtain the following equation.[5]

$$\frac{4t^3}{y^2} + \frac{2t^{3/2}}{y} - y^4 + y^2 = 0. \tag{3.44}$$

Therefore, the low energy superpotential is give by

$$W_{\rm exact} = \frac{m^2}{g}\left[-\frac{1}{2} + 4t^{3/2} + 2(t^{3/2})^2 - 2(t^{3/2})^3 + 4(t^{3/2})^4 - 21(t^{3/2})^5 + \mathcal{O}\left((t^{3/2})^6\right)\right]. \tag{3.45}$$

By comparing this result with (3.21) in the classical limit, we find $N_1 = 1$ and $N_0 = 1$. Thus this low-energy superpotential corresponds to the gauge symmetry breaking Sp(4) $\to$ Sp(2) $\times$ U(1).

[5] Here we remark that this equation is same as that of SO(8) $\to$ SO(6) $\times$ U(1). This is consistent with the analysis in the dual geometry. The effective superpotential (2.21) of $N_0 = 1$, $N_1 = 1$ for negative orientifold plane charge is equal to that of $N_0 = 3$, $N_1 = 1$ for positive orientifold plane charge. Therefore these theories should have same low-energy superpotentials $W_{\rm exact}$.

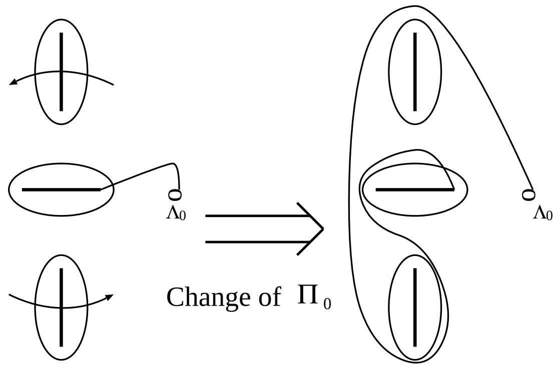

Figure 2: The change of Π_0 under the $a_1 \to e^{2\pi i} a_1$.

4. Analysis of dual geometry

As reviewed in section 2, the 3-form fluxes through the special lagrangian 3-cycles generate the superpotential and it can be identified with the effective superpotential in the dual gauge theory. In this section, we will calculate the periods S_i's and Π_i's for the case of $n = 1$. To find effective superpotential for the glueball superfields, we will compute the dual periods Π_i's in terms of the periods S_i's.

4.1 Monodoromy analysis

As in [12], we will express the period of the dual cycles in terms of the period of S^3's. In this subsection, we will discuss their logarithmic terms from monodromy analysis. First, we consider the transformation, $\Lambda_0 \to e^{2\pi i}\Lambda_0$. Under this transformation, the period Π_i changes by,

$$\Delta\Pi_i = -2\left(S_0 + 2S_1\right). \tag{4.1}$$

Therefore, we have the logarithmic dependence on Λ_0 as,

$$\Pi_i = -\frac{2}{2\pi i}\left(S_0 + 2S_1\right)\log\Lambda_0 + \cdots . \tag{4.2}$$

Next we consider the transformation, $\mu_i \to e^{2\pi i}\mu_i$ ($\mu_i \equiv S_i/2W''(a_i)$). Under this transformation, Π_i changes by

$$\Delta\Pi_i = S_i\,, \tag{4.3}$$

so that,

$$\Pi_i = \frac{S_i}{2\pi i}\log\frac{S_i}{2W''(a_i)}\,. \tag{4.4}$$

Finally, we will consider the transformation, $a_1 \to e^{2\pi i}a_1$(see figure 2). Under this transformation Π_0, Π_1 change respectively by

$$\Delta\Pi_0 = 2\left(2S_1\right), \qquad \Delta\Pi_1 = 2\left(S_0 + S_1\right). \tag{4.5}$$

Collecting these results, we have the following monodromy contributions.

$$\begin{aligned}\Pi_0 &= \frac{W(\Lambda_0)-W(0)}{\pi i} - \frac{1}{\pi i}(S_0+2S_1)\log\Lambda_0 + \\ &\quad + \frac{S_0}{2\pi i}\log\frac{S_0}{2W''(0)} + \frac{4}{2\pi i}S_1\log a_1 + \cdots, \\ \Pi_1 &= \frac{W(i\Lambda_0)-W(ia_1)}{\pi i} - \frac{1}{\pi i}(S_0+2S_1)\log\Lambda_0 + \\ &\quad + \frac{S_1}{2\pi i}\log\frac{S_1}{2W''(ia_1)} + \frac{2}{2\pi i}(S_0+S_1)\log a_1 + \cdots . \end{aligned} \tag{4.6}$$

Plugging these terms into (2.21), we obtain the effective superpotential, which is derived from naive integrating in [11][12].

4.2 Effective superpotential

In the previous section, we obtained the logarismic contribution of the dual periods Π_i's from the monodromy analysis. In this subsection we will compute the remaining terms. The explicit computation of Π_0 and Π_1 can be found in appendix up to the 4th order in S_i.

$$\begin{aligned}-\pi i\Pi_0 &= -W(\Lambda_0)+W(0)-(S_0+2S_1)\log\Lambda_0+\frac{S_0}{2}\log\frac{S_0}{2W''(0)}+2S_1\log a_1+ \\ &\quad + ga_1^4\Bigg[-2(x+y)\log\left(\frac{\Lambda_0}{a_1}\right)+x(\log x-1)+4xy-\frac{3}{2}x^2-\frac{1}{2}y^2+\frac{9}{2}x^3- \\ &\quad\quad -21x^2y+9xy^2-\frac{1}{2}y^3\frac{45}{2}x^4+\frac{466}{3}x^3-y+\frac{76}{3}xy^3-131x^2y^2-\frac{5}{6}y^4\Bigg]+\cdots, \end{aligned} \tag{4.7}$$

$$\begin{aligned}-\pi i\Pi_1 &= -W(i\Lambda_0)+W(ia_1)-(S_0+2S_1)\log\Lambda_0+\frac{S_1}{2}\log\frac{S_1}{2W''(ia_1)}+2(S_0+S_1)\log a_1+ \\ &\quad + ga_1^4\Bigg[-2(x+y)\log\left(\frac{\Lambda_0}{a_1}\right)+\frac{y}{2}(\log y-1)+2x^2-xy-7x^3+9x^2y-\frac{6}{4}xy^2+ \\ &\quad\quad +\frac{233}{6}x^4-\frac{262}{3}x^3y+\frac{152}{4}x^2y^2-\frac{10}{3}xy^3\Bigg]+\cdots, \end{aligned} \tag{4.8}$$

where $x \equiv S_0/2ga_1^4$, $y \equiv S_1/ga_1^4$. In this expression, the cut off Λ_0 is combined with the bare coupling α to form the gauge theory scale Λ of the underlying $\mathcal{N}=2$ Yang-Mills theory [12][17].

The effective superpotential is given by (2.21). Since the exact low energy superpotential is obtained by integrating out the glueball superfields S_0, S_1, we need to solve the equations $\partial W_{\rm eff}/\partial x = 0$, $\partial W_{\rm eff}/\partial y = 0$. In the leading order, these equations are solved as,

$$x = T^{N_1/2}, \qquad y = T^{N_0/2-1}, \qquad T \equiv \left(\frac{\Lambda}{a_1}\right)^{\frac{4(N_0+2N_1-2)}{N_1(N_0-2)}} . \tag{4.9}$$

Using T as the expansion parameter, the low energy superpotential $W_{\rm low}$ can be evaluated. Now we will compute some examples that correspond to the examples in the previous section.

$\mathrm{SO}(6) \to \mathrm{SO}(4) \times \mathrm{U}(1)$. As the first example, we consider the case $\mathrm{SO}(6) \to \mathrm{SO}(4) \times \mathrm{U}(1)$. This gauge theory is realized by taking values as, $N_0 = 4$, $N_1 = 1$ and $T = g^2\Lambda^4/m^2 \equiv t^2$ The effective superpotential is obtained by plugging these values into the expression (2.21).

$$W_{\mathrm{low}} = \frac{m^2}{g}\left[-\frac{1}{2} + 2T + \mathcal{O}(T^5)\right]. \tag{4.10}$$

In the calculation of this low-energy superpotential, some miraculous cancellation happens and this superpotential coincides with the confining phase superpotential (3.27). Thus the large N duality is proved for this case up to the order $\mathcal{O}(T^5)$.

Case 2: Splitting of $\mathrm{SO}(8)$. As a next example, we will consider the case which corresponds to the splitting of the gauge group $\mathrm{SO}(8)$. In this case, we consider the superpotentials generated by the fluxes of $N = N_0 + 2N_1 = 8$ D5-branes and a O^--plane. To compare with the results of gauge theory, we will consider the following two breaking patterns.

$\mathrm{SO}(8) \to \mathrm{SO}(6) \times \mathrm{U}(1)$. First, we consider the case $\mathrm{SO}(8) \to \mathrm{SO}(6) \times \mathrm{U}(1)$. This breaking is realized by choosing $N_0 = 6$, $N_1 = 1$ and $T = t^{3/2}$. Thus effective superpotential is given by,

$$W_{\mathrm{low}} = \frac{m^2}{g}\left[-\frac{1}{2} + 4T + 2T^2 - 2T^3 + 4T^4 + \mathcal{O}(T^5)\right]. \tag{4.11}$$

This superpotential coincides with the exact superpotential (3.36) in the gauge theory analysis up to $\mathcal{O}(T^5)$.

$\mathrm{SO}(8) \to \mathrm{SO}(4) \times \mathrm{U}(2)$. As another splitting, we consider the case $\mathrm{SO}(8) \to \mathrm{SO}(4) \times \mathrm{U}(2)$. This breaking corresponds to the case $N_0 = 4$, $N_1 = 2$ and $T = t^{\frac{3}{2}}$. Thus effective superpotential is

$$W_{\mathrm{low}} = \frac{m^2}{g}\left[-1 + 4T^2 - 8T^4 + 64T^6 - 768T^8 + \mathcal{O}(T^{10})\right]. \tag{4.12}$$

This superpotential also coincides with the confining phase superpotential (3.38). Thus we examined the large N duality for both of the breaking patterns in the gauge theory.

Case 3: $\mathrm{Sp}(4) \to \mathrm{Sp}(2) \times \mathrm{U}(1)$. Finally we consider a example of Sp gauge theory. In this case, we have to choose the plus sign in (2.19) and set $N_0 = 2, N_1 = 1$. The low-energy superpotential is

$$W_{\mathrm{low}} = -\frac{m^2}{g}\left[-\frac{1}{2} + 4T + 2T^2 - 2T^3 + 4T^4 + \mathcal{O}(T^5)\right]. \tag{4.13}$$

This superpotential coincides with the confining phase superpotential (3.45) Thus we examined the large N duality for $\mathrm{Sp}(2N)$ gauge theory.

Acknowledgments

The authors thank to Norisuke Sakai for reading this manuscript and giving us useful comments. The authors are obliged especially to Katsushi Ito for stimulus discussions and useful suggestions. Y.O. would like to thank M. Eto for continuous encouragement. H.F. is supported by JSPS research fellowship for young scientists.

JHEP02(2003)028

A. Computation of periods

In this appendix we will show the explicit computation of the periods Π_0, Π_1 in (2.21). As discussed in section 2, the effective one-form is written as follows,

$$2dx\sqrt{W'^2(x)+f_2(x)} = 2dxg\sqrt{(x^2-0^{+2})(x^2+x_1^2)(x^2+x_2^2)}\,. \tag{A.1}$$

Comparing the coefficient of x^3 on both sides in the above equation, we obtain the following relation.

$$\frac{m}{g} = \frac{1}{2}\left(-\Delta_0^2+x_1^2+x_2^2\right) = a_1^2\,. \tag{A.2}$$

Here we define new variables given by,

$$\begin{aligned}\Delta_0 &\equiv 0^+ = -0^-\,, \qquad \Delta_1 \equiv \frac{1}{2}(x_2-x_1)\,,\\ Q &\equiv \frac{1}{2}(x_2+x_1)\,, \qquad w \equiv \frac{\Delta_0}{Q}\,, \qquad u \equiv \frac{\Delta_1}{Q}\,.\end{aligned} \tag{A.3}$$

Since f_2 is considered as a small perturbation, Δ_0, Δ_1 and Q satisfy

$$|\Delta_0| \sim |\Delta_1| \ll |Q|\,. \tag{A.4}$$

Under this relation, we can expand the periods S_0 and S_1 in powers of w and u.

$$\begin{aligned}S_0 &= \frac{g}{\pi i}\int_{-\Delta_0}^{\Delta_0}\sqrt{(x^2-\Delta_0^2)(x^2+x_1^2)(x^2+x_2^2)}\;dx\\ &= \frac{g}{2}Q^2w^2\left[1-u^2+\frac{w^2}{4}+\frac{w^2u^2}{2}+\frac{w^2u^4}{2}-\frac{u^2w^4}{4}+\mathcal{O}\left((uw)^7\right)\right], \qquad \text{(A.5)}\\ S_1 &= \frac{g}{\pi i}\int_{x_1}^{x_2}\sqrt{(x^2-\Delta_0^2)(x^2+x_1^2)(x^2+x_2^2)}\;dx\\ &= gQ^4u^2\left[1+\frac{1}{2}w^2-\frac{1}{8}w^4-\frac{1}{8}w^4u^2+\frac{1}{16}w^6+\mathcal{O}\left((uw)^7\right)\right]. \qquad \text{(A.6)}\end{aligned}$$

Next we will compute the dual periods Π_i's. In the computation, we will discard any contributions of $\mathcal{O}(\Lambda_0^{-1})$, since Λ_0 is introduced as cut-off of infinite volume of dual three cycles.

$$\begin{aligned}\frac{\pi i}{g}\Pi_0 &= \int_{\Delta_0}^{\Lambda_0}\sqrt{(x^2-\Delta_0^2)(x^2+x_1^2)(x^2+x_2^2)}\;dx\\ &= \frac{1}{4}(\Lambda_0)^4+\frac{1}{2}(\Lambda_0)^2Q^2\left(1+u^2-\frac{1}{2}w^2\right)-Q^4\log(2\Lambda_0)\left(\frac{w^2}{2}+\frac{w^2u^2}{2}+\frac{w^4}{8}+2u^2\right)+\\ &\quad+\frac{Q^4}{32}w^4-\frac{Q^4w^2}{4}\left(1+u^2\right)+\sum_{n=2}^{\infty}c_n\frac{(-1)^{n-2}}{(n-2)!}(4Q^2u)^nG^{(n-2)}(a)\Big|_{a=0}\,, \qquad \text{(A.7)}\end{aligned}$$

where c_n are the coefficient of the power expansion of $\sqrt{1+x}$. The computation for Π_1 is obtained in a similar way.

$$\begin{aligned}
\frac{\pi i}{g}\Pi_1 &= \int_{x_2}^{\Lambda_0} \sqrt{(x^2+\Delta_0^2)(x^2+x_1^2)(x^2+x_2^2)}dx \\
&= \frac{\Lambda_0^6}{4} + \Lambda_0^3 Q^2 \left(-\frac{1+u^2}{2} + \frac{w^2}{4}\right) + Q^4 \log(4\Lambda^3) \left(-u^2 - \frac{(1+u^2)w^2}{4}\right) - \\
&\quad -Q^4 \frac{(1+u^2)w^2}{4} + Q^4 \left(u^2 + \frac{w^2(1+u^2)}{4}\right) \log(4Q^2u^2) + Q^4 \frac{1+u^4}{4} - \\
&\quad -\frac{1}{2}\sum_{n=1}^{\infty} c_n (-1)^{(n)} \frac{w^{2n}Q^{2n}}{(n-1)!} H^{(n-1)}(a)\bigg|_{a=0} . \qquad \text{(A.8)}
\end{aligned}$$

In the above expansion, we used the following generating functions

$$\begin{aligned}
G(a) &\equiv -\sqrt{\frac{x_1^2+\Delta_0^2+a}{x_1^2+a}} \log\left(\sqrt{x_1^2+\Delta_0^2+a} + \sqrt{x_1^2+a} - \log\Delta_0\right), \\
H(a) &\equiv -a\left(\log(4\Lambda_0^3) - \log(4Q^2u)\right) + \sqrt{(x_1^2+a)(x_2^2+a)} \times \\
&\quad \times \log\Bigg((\Lambda_0^3+a)(x_1^2-x_2^2)/\{-(x_1^2+x_2^2+2a)(\Lambda_0^3+a) + \\
&\qquad + 2(x_1^2+a)(x_2^2+a) + 2\sqrt{(x_1^2+a)(x_2^2+a)(\Lambda_0^3-x_1^2)(\Lambda_0^3-x_2^2)}\}\Bigg) .
\end{aligned}$$

The dual periods Π_0 and Π_1 are expressed in terms of the periods S_0 and S_1 by comparing order by order in w and u in the expressions (A.5)$\sim$(A.8).

$$\begin{aligned}
-\pi i \Pi_0 &= W(\Lambda_0) + W(0)\,(S_0+2S_1)\log\Lambda_0 - +\frac{S_0}{2}\log\frac{S_0}{2W''(0)} + 2S_1\log a_1 - \frac{S_0}{2} + \\
&\quad + \frac{1}{4ga_1^4}\left(8S_0S_1 - \frac{3}{2}S_0^2 - 2S_1^2\right) + \frac{1}{8(ga_1^4)^2}\left(\frac{9}{2}S_0^3 - 42S_0^2S_1 + 36S_0S_1^2 - 4S_1^3\right) + \\
&\quad + \frac{1}{16(ga_1^4)^3}\left(-\frac{45}{2}S_0^4 + \frac{932}{3}S_0^3S_1 + \frac{608}{3}S_0S_1^3 - 524S_0^2S_1^2 - \frac{40}{3}S_1^4\right) + \cdots, \qquad \text{(A.9)} \\
-\pi i \Pi_1 &= W(i\Lambda_0) + W(ia_1) - (S_0+2S_1)\log -\frac{\Lambda_0}{a_1} - S_1\log a_1 + \frac{S_1}{2}\log -\frac{S_1}{m} - \frac{S_1}{2} + \\
&\quad + \frac{1}{4ga_1^4}\left(2S_0^2 - 2S_0S_1\right) + \frac{1}{8(ga_1^4)^2}\left(-7S_0^3 + 18S_0^2S_1 - 6S_0S_1^2\right) + \\
&\quad + \frac{1}{16(ga_1^4)^3}\left(\frac{233}{6}S_0^4 - \frac{524}{3}S_0^3S_1 + 152S_0^2S_1^2 - \frac{80}{3}S_0S_3^3\right) + \cdots . \qquad \text{(A.10)}
\end{aligned}$$

References

[1] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.

[2] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200]; *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];

S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[3] I.R. Klebanov and E. Witten, *Superconformal field theory on threebranes at a Calabi-Yau singularity*, *Nucl. Phys.* **B 536** (1998) 199 [hep-th/9807080].

[4] I.R. Klebanov and N.A. Nekrasov, *Gravity duals of fractional branes and logarithmic RG flow*, *Nucl. Phys.* **B 574** (2000) 263 [hep-th/9911096];
I.R. Klebanov and A.A. Tseytlin, *Gravity duals of supersymmetric* $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ *gauge theories*, *Nucl. Phys.* **B 578** (2000) 123 [hep-th/0002159];
I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [hep-th/0007191].

[5] R. Gopakumar and C. Vafa, *Topological gravity as large-N topological gauge theory*, *Adv. Theor. Math. Phys.* **2** (1998) 413 [hep-th/9802016]; *On the gauge theory/geometry correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1415 [hep-th/9811131]; *M-theory and topological strings, I*, hep-th/9809187;
M-theory and topological strings, II, hep-th/9812127;
H. Ooguri and C. Vafa, *Knot invariants and topological strings*, *Nucl. Phys.* **B 577** (2000) 419 [hep-th/9912123];
H. Ooguri and C. Vafa, *Worldsheet derivation of a large-N duality*, *Nucl. Phys.* **B 641** (2002) 3 [hep-th/0205297].

[6] C. Vafa, *Superstrings and topological strings at large-N*, *J. Math. Phys.* **42** (2001) 2798 [hep-th/0008142].

[7] M. Atiyah, J.M. Maldacena and C. Vafa, *An M-theory flop as a large-N duality*, *J. Math. Phys.* **42** (2001) 3209 [hep-th/0011256].

[8] M. Atiyah and E. Witten, *M-theory dynamics on a manifold of G_2 holonomy*, *Adv. Theor. Math. Phys.* **6** (2003) 1 [hep-th/0107177];
H. Ita, Y. Oz and T. Sakai, *Comments on M-theory dynamics on G_2 holonomy manifolds*, *J. High Energy Phys.* **04** (2002) 001 [hep-th/0203052];
T. Friedmann, *On the quantum moduli space of M-theory compactifications*, *Nucl. Phys.* **B 635** (2002) 384 [hep-th/0203256].

[9] M. Aganagic, A. Klemm and C. Vafa, *Disk instantons, mirror symmetry and the duality web*, *Z. Naturforsch.* **A57** (2002) 1 [hep-th/0105045].

[10] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[11] G. Veneziano and S. Yankielowicz, *An effective lagrangian for the pure $N = 1$ supersymmetric Yang-Mills theory*, *Phys. Lett.* **B 113** (1982) 321;
T.R. Taylor, G. Veneziano and S. Yankielowicz, *Supersymmetric QCD and its massless limit: an effective lagrangian analysis*, *Nucl. Phys.* **B 218** (1983) 493.

[12] F. Cachazo, K.A. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067].

[13] F. Cachazo, S. Katz and C. Vafa, *Geometric transitions and $N = 1$ quiver theories*, hep-th/0108120;
F. Cachazo, B. Fiol, K.A. Intriligator, S. Katz and C. Vafa, *A geometric unification of dualities*, *Nucl. Phys.* **B 628** (2002) 3 [hep-th/0110028];
B. Fiol, *Duality cascades and duality walls*, *J. High Energy Phys.* **07** (2002) 058 [hep-th/0205155].

[14] K. Dasgupta, K. Oh and R. Tatar, *Geometric transition, large-N dualities and MQCD dynamics*, *Nucl. Phys.* **B 610** (2001) 331 [hep-th/0105066]; *Open/closed string dualities and Seiberg duality from geometric transitions in M-theory*, *J. High Energy Phys.* **08** (2002) 026 [hep-th/0106040];
K. Dasgupta, K.-h. Oh, J. Park and R. Tatar, *Geometric transition versus cascading solution*, *J. High Energy Phys.* **01** (2002) 031 [hep-th/0110050];
K.-H. Oh and R. Tatar, *Duality and confinement in $N = 1$ supersymmetric theories from geometric transitions*, *Adv. Theor. Math. Phys.* **6** (2003) 141 [hep-th/0112040]; *Renormalization group flows on D3-branes at an orbifolded conifold*, *J. High Energy Phys.* **05** (2000) 030 [hep-th/0003183].

[15] S. Sinha and C. Vafa, SO *and* Sp *Chern-Simons at large-N*, hep-th/0012136.

[16] B. Acharya, M. Aganagic, K. Hori and C. Vafa, *Orientifolds, mirror symmetry and superpotentials*, hep-th/0202208.

[17] J.D. Edelstein, K. Oh and R. Tatar, *Orientifold, geometric transition and large-N duality for* SO/Sp *gauge theories*, *J. High Energy Phys.* **05** (2001) 009 [hep-th/0104037].

[18] S. Imai and T. Yokono, *Comments on orientifold projection in the conifold and* SO × USp *duality cascade*, *Phys. Rev.* **D 65** (2002) 066007 [hep-th/0110209].

[19] J. Gomis, *On SUSY breaking and χSB from string duals*, *Nucl. Phys.* **B 624** (2002) 181 [hep-th/0111060].

[20] S. Kachru, S. Katz, A.E. Lawrence and J. McGreevy, *Open string instantons and superpotentials*, *Phys. Rev.* **D 62** (2000) 026001 [hep-th/9912151]; *Mirror symmetry for open strings*, *Phys. Rev.* **D 62** (2000) 126005 [hep-th/0006047];
I. Brunner, M.R. Douglas, A.E. Lawrence and C. Romelsberger, *D-branes on the quintic*, *J. High Energy Phys.* **08** (2000) 015 [hep-th/9906200].

[21] S. Elitzur, A. Forge, A. Giveon, K.A. Intriligator and E. Rabinovici, *Massless monopoles via confining phase superpotentials*, *Phys. Lett.* **B 379** (1996) 121 [hep-th/9603051].

[22] S. Terashima and S.-K. Yang, *Confining phase of $N = 1$ supersymmetric gauge theories and $N = 2$ massless solitons*, *Phys. Lett.* **B 391** (1997) 107 [hep-th/9607151]; *Ade confining phase superpotentials*, *Nucl. Phys.* **B 519** (1998) 453 [hep-th/9706076];
S. Terashima, *Supersymmetric gauge theories with classical groups via M-theory fivebrane*, *Nucl. Phys.* **B 526** (1998) 163 [hep-th/9712172].

[23] P. Candelas and X.C. de la Ossa, *Comments on conifolds*, *Nucl. Phys.* **B 342** (1990) 246.

[24] A. Strominger, *Massless black holes and conifolds in string theory*, *Nucl. Phys.* **B 451** (1995) 96 [hep-th/9504090].

[25] I. Antoniadis, H. Partouche and T.R. Taylor, *Spontaneous breaking of $N = 2$ global supersymmetry*, *Phys. Lett.* **B 372** (1996) 83 [hep-th/9512006];

S. Ferrara, L. Girardello and M. Porrati, *Spontaneous breaking of $N = 2$ to $N = 1$ in rigid and local supersymmetric theories*, *Phys. Lett.* **B 376** (1996) 275 [hep-th/9512180];
I. Antoniadis and T.R. Taylor, *Dual $N = 2$ SUSY breaking*, *Fortschr. Phys.* **44** (1996) 487 [hep-th/9604062].

[26] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070];
S. Gukov, *Solitons, superpotentials and calibrations*, *Nucl. Phys.* **B 574** (2000) 169 [hep-th/9911011].

[27] N. Seiberg and E. Witten, *Electric-magnetic duality, monopole condensation and confinement in $N = 2$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 426** (1994) 19 [hep-th/9407087].

[28] R.G. Leigh and M.J. Strassler, *Duality of* $\mathrm{Sp}(2N_c)$ *and* $\mathrm{SO}(N_c)$ *supersymmetric gauge theories with adjoint matter*, *Phys. Lett.* **B 356** (1995) 492 [hep-th/9505088].

[29] P.C. Argyres and A.D. Shapere, *The vacuum structure of $N = 2$ superqcd with classical gauge groups*, *Nucl. Phys.* **B 461** (1996) 437 [hep-th/9509175].

[30] U.H.Danielsson and B.Sundborg, *The moduli space and monodromies of $N = 2$ supersymmetric* $\mathrm{SO}(2r + 1)$ *Yang-Mills theory*, *Phys. Lett.* **B 358** (1995) 273 [hep-th/95044102]
A. Brandhuber and K. Landsteiner, *On the monodromies of $N = 2$ supersymmetric Yang-Mills theory with gauge group* $\mathrm{SO}(2N)$, *Phys. Lett.* **B 358** (1995) 73 [hep-th/9507008].

[31] C.-H. Ahn, *Confining phase of $N = 1$* $\mathrm{Sp}(N_c)$ *gauge theories via M-theory fivebrane*, *Phys. Lett.* **B 426** (1998) 306 [hep-th/9712149].

[32] C.-H. Ahn, K. Oh and R. Tatar, *M-theory fivebrane and confining phase of $N = 1$* $\mathrm{SO}(N_c)$ *gauge theories*, *J. Geom. Phys.* **28** (1998) 163 [hep-th/9712005].

[33] M.R. Douglas and S.H. Shenker, *Dynamics of* $\mathrm{SU}(N)$ *supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 271 [hep-th/9503163].

[34] S.K. Ashok, R. Corrado, N. Halmagyi, K.D. Kennaway and C. Romelsberger, *Unoriented strings, loop equations and $N = 1$ superpotentials from matrix models*, hep-th/0211291.

JHEP02(2003)028